

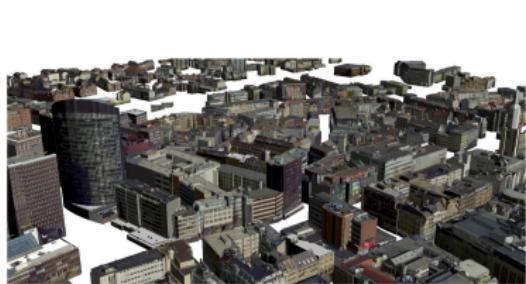
Beautification of City Models based on Mixed Integer Linear Programming

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OR2018, 12.09.2018

CityGML: Virtual 3D city models



Google Earth/Google Maps 3D

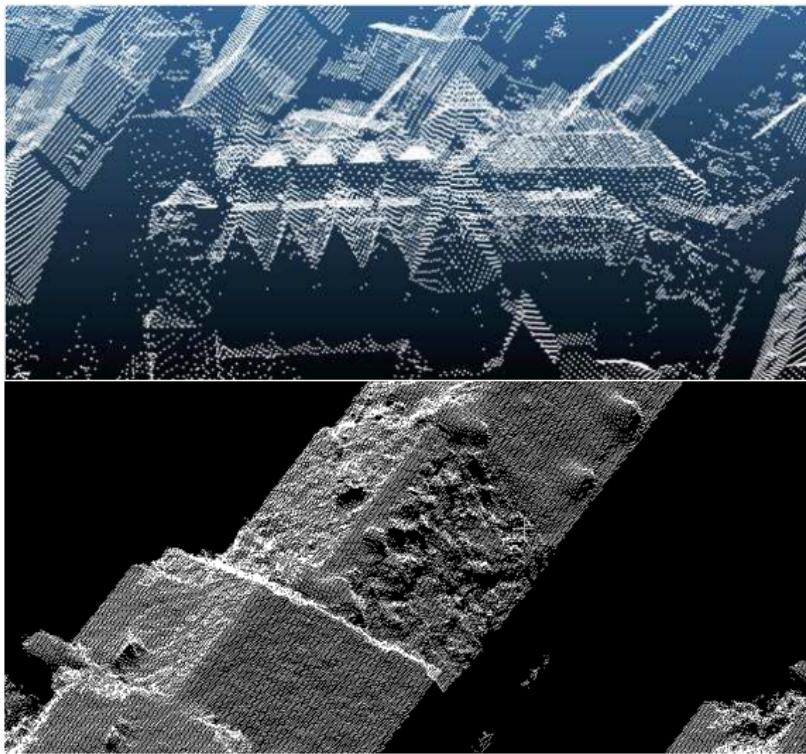


Some applications of city models

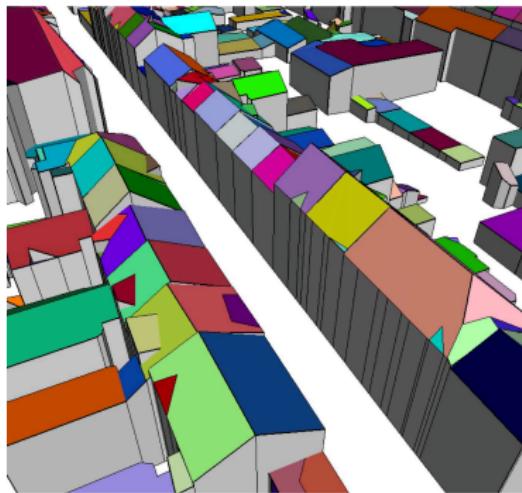
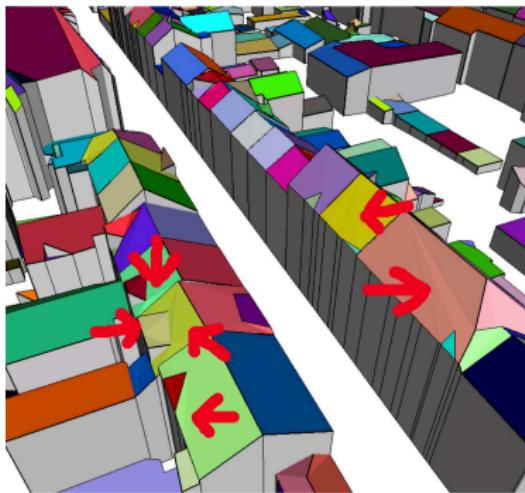


- Building Information Modeling (BIM)
- emergency response planning
- solar potential analysis
- urban planning, cadastre
- visualization for navigation and routing
- energy demand estimation
- virtual tours, tourism
- facility management
- flooding simulation
- visibility analysis
- shadow estimation
- noise propagation
- estimation of floor space and population

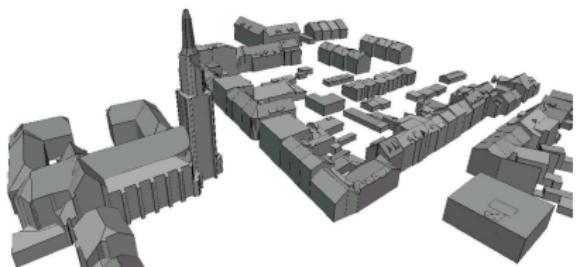
Laser scanning and photogrammetric point clouds



Previous applications of LP: Planarization of roof facets



Previous applications of MIP: Point cloud registration

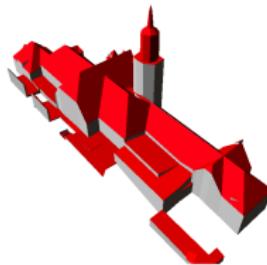
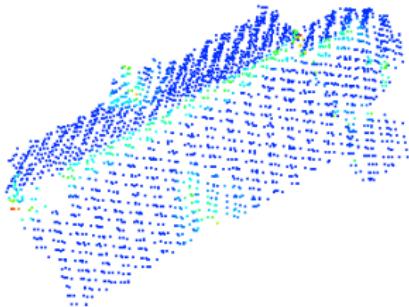


Model vs. data based method

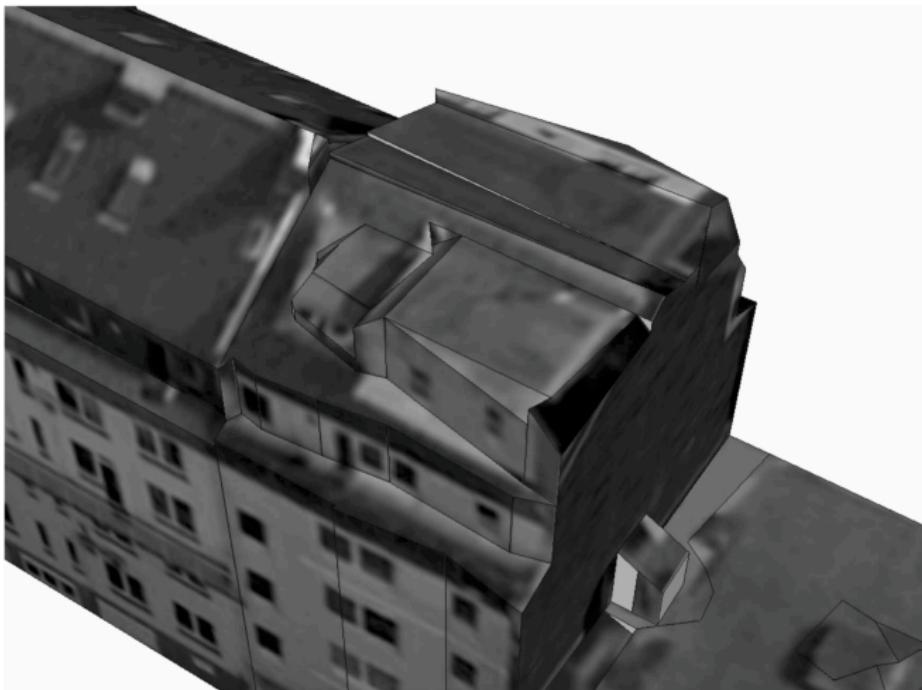
- Model based North Rhine Westphalian city model (©Geobasis NRW 2016)



- Our institute's data based model

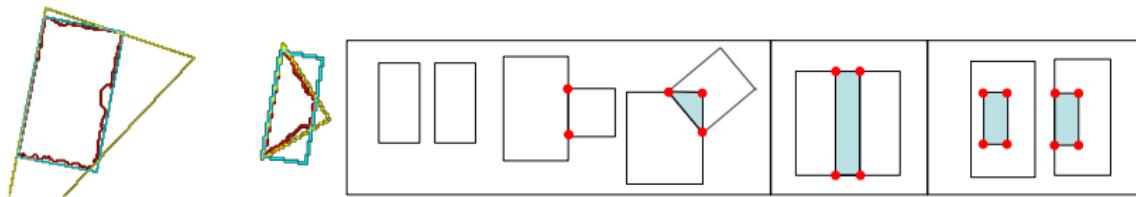


A Problem with data based models: noisy step edges

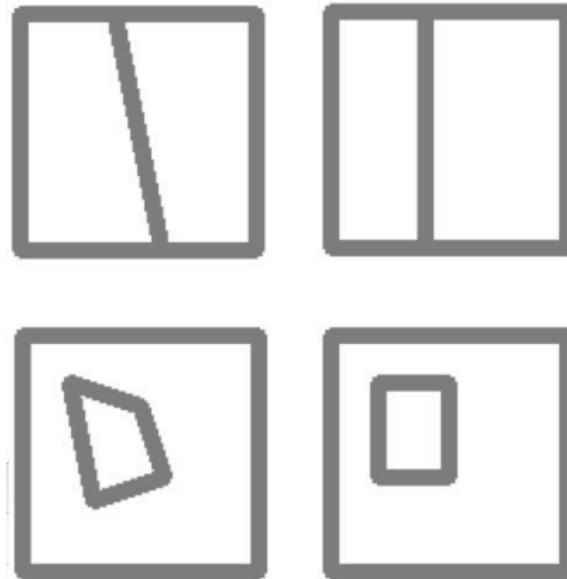


Solution: Beautification - elementary approach

- Project roof edges to the x-y-plane.
- Compute convex hulls of roof facet's boundary polygons
- Compare their area with the area of their smallest enclosing triangles, circles and rectangles
- If the rectangle area is closest to the convex hull's area and the difference is below a threshold value, then the contour might be rectangular.
- Replace contour with rectangle if additional heuristic conditions are fulfilled.

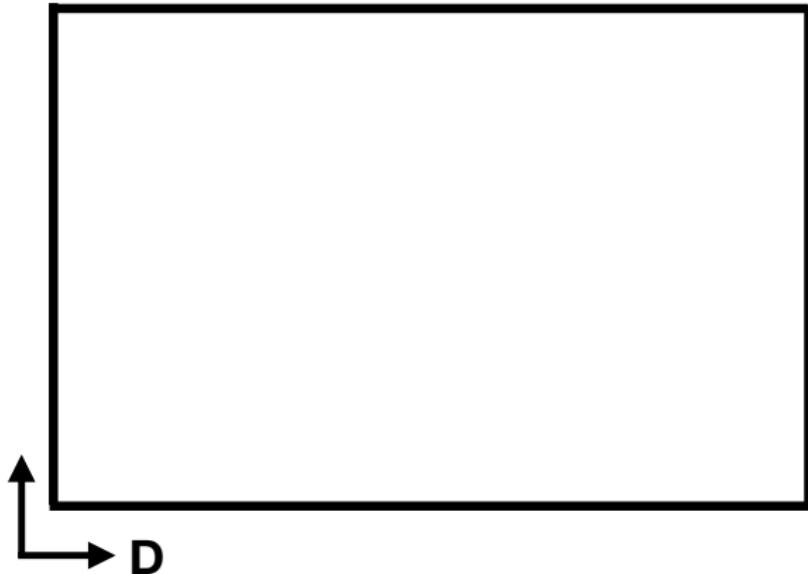


Solution: Beautification - optimization approach



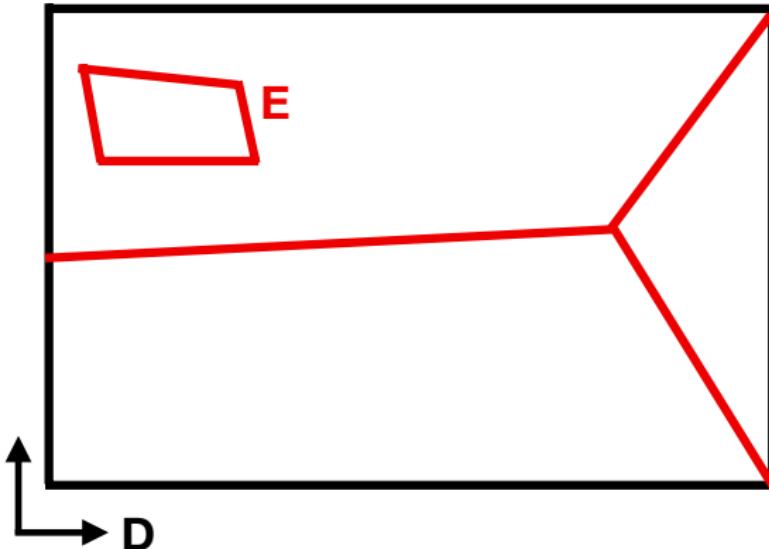
- Project roof edges to the x-y-plane.
- Detect connected components of 2D graph
- Use a MIP to maximize the number of orthogonal edges for each connected component.

Notations (1)



- D is a set of normalized 2D direction vectors that are parallel or orthogonal to significant cadastral footprint edges.

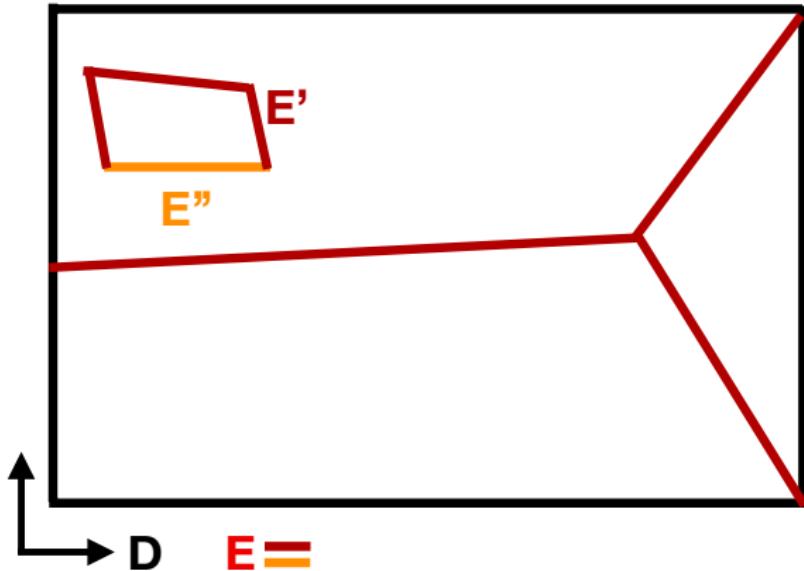
Notations (2)



The figure shows two connected components that are handled separately.

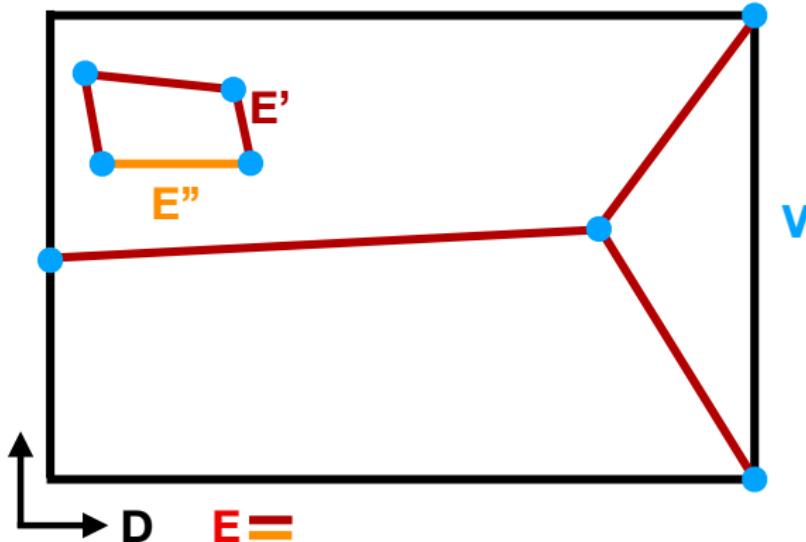
- For each component, E is the set of all non-trivial 2D edges $e = (e_1, e_2) = ((e_1.x, e_1.y), (e_2.x, e_2.y))$ that do not lie completely on the cadastral footprint.

Notations (3)



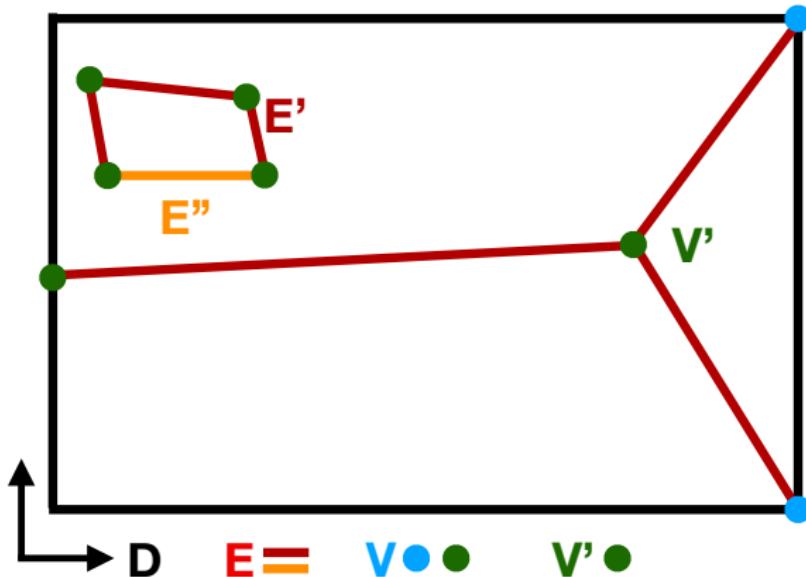
- $E = E' \cup E''$, edges in E' are allowed to change their orientation, whereas edges in E'' have to keep their direction that already is orthogonal to a vector of D

Notations (4)



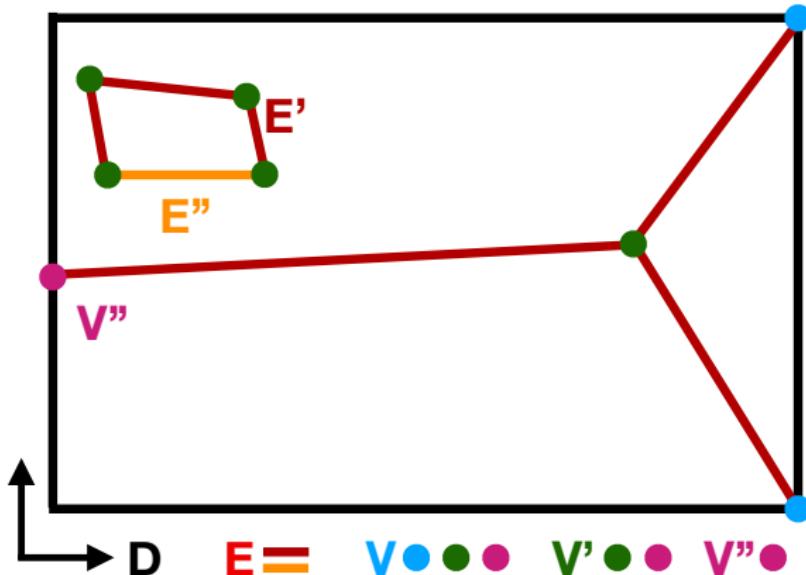
- V denotes the set of vertices belonging to edges in E

Notations (5)



- $V' \subset V$ is the set of vertices for which we allow position changes.

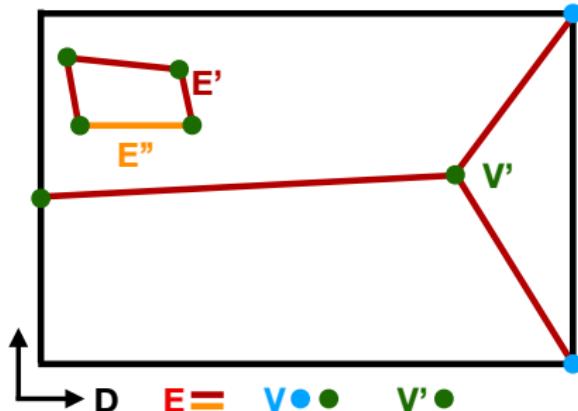
Notations (6)



- $V'' \subset V'$ is the set of vertices that lie on a footprint edge.

Variables

For $v \in V$, float variables x_v^+ , x_v^- and y_v^+ , y_v^- represent non-negative changes. New coordinates are $v.x + x_v^+ - x_v^-$, $v.y + y_v^+ - y_v^-$. For $v \in V \setminus V'$ their values are fixed set to zero.



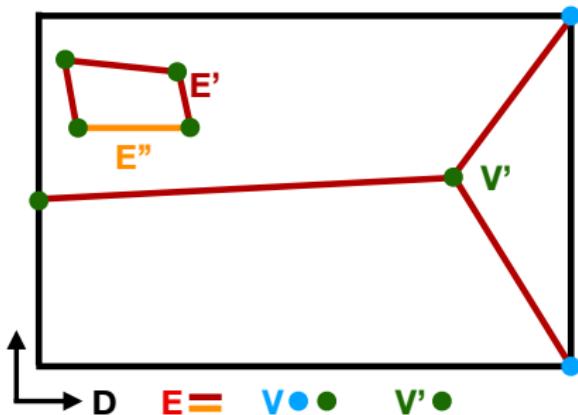
We introduce binary variables $x_{e,d}$ that indicate, if an edge $e \in E'$ becomes orthogonal to a direction vector $d \in D$:

$$x_{e,d} = \begin{cases} 1 & : e \text{ is transformed to become orthogonal to } d \\ 0 & : \text{else} \end{cases}$$

Objective function

- Primary goal: find a maximum number of orthogonal edges
- Secondary goal: minimize coordinate changes.

$$\text{maximize} \left(\sum_{e \in E'} \sum_{d \in D} x_{e,d} \right) - \frac{1}{8 \cdot |V'| \cdot \varepsilon} \sum_{v \in V'} (x_v^+ + x_v^- + y_v^+ + y_v^-).$$



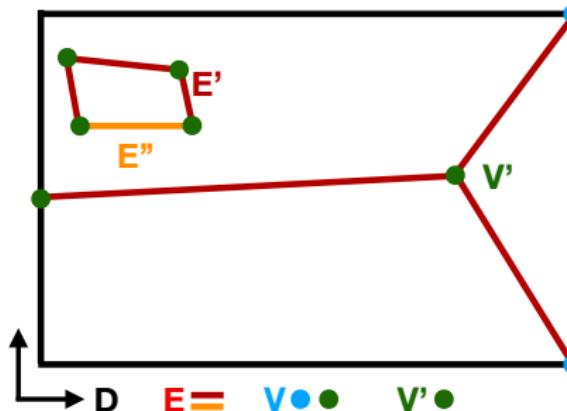
Restrictions (1)

We do not change vertices on the cadastral footprint and intersection points of ridge lines:

$$x_v^+ = x_v^- = y_v^+ = y_v^- = 0 \text{ for all } v \in V \setminus V'.$$

Other vertices can be moved but only within a threshold distance $\varepsilon > 0$:

$$0 \leq x_v^+, x_v^-, y_v^+, y_v^- < \varepsilon \text{ for all } v \in V'.$$

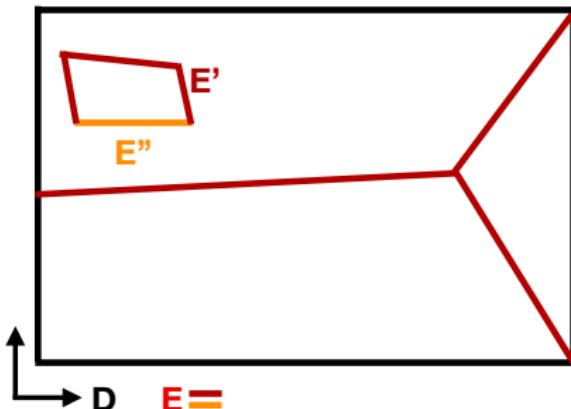


Restrictions (2)

Modified edges $e \in E'$ have to be orthogonal to direction $d \in D$ if $x_{e,d} = 1$:

$$\begin{aligned}-M \cdot (1 - x_{e,d}) &\leq (e_2 \cdot x + x_{e_2}^+ - x_{e_2}^- - e_1 \cdot x - x_{e_1}^+ + x_{e_1}^-) \cdot d.x \\&\quad + (e_2 \cdot y + y_{e_2}^+ - y_{e_2}^- - e_1 \cdot y - y_{e_1}^+ + y_{e_1}^-) \cdot d.y \\&\leq M \cdot (1 - x_{e,d}),\end{aligned}$$

Constant M is larger than the longest occurring edge.



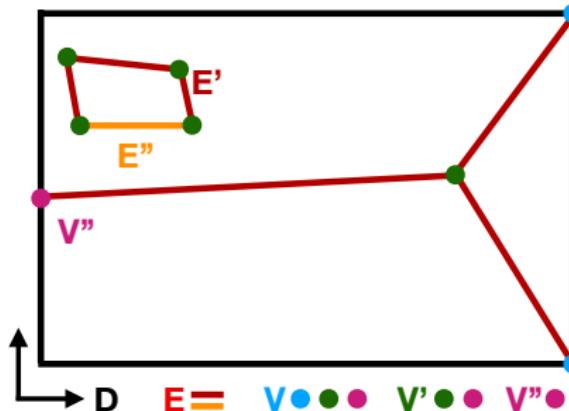
Restrictions (3)

Vertices in V'' are only allowed to change their positions so that they stay on the one footprint edge that they are positioned on:
Let $0 \leq r_v \leq 1$ and

$$x_v^+ - x_v^- - (b_v.x - a_v.x) \cdot r_v = a_v.x - v.x$$

$$y_v^+ - y_v^- - (b_v.y - a_v.y) \cdot r_v = a_v.y - v.y,$$

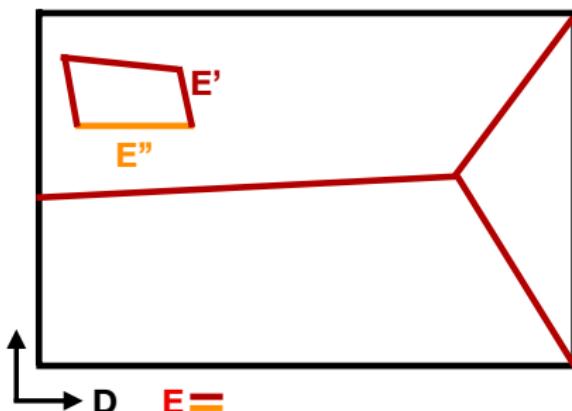
where a_v and b_v denote the 2D vertices of the corresponding footprint edge.



Restrictions (4)

We also keep the orientation of edges that are orthogonal to a footprint direction $d \in D$ from the beginning: For all $e \in E''$ let

$$(x_{e_2}^+ - x_{e_2}^- - x_{e_1}^+ + x_{e_1}^-) \cdot (e_1.y - e_2.y) \\ + (y_{e_2}^+ - y_{e_2}^- - y_{e_1}^+ + y_{e_1}^-) \cdot (e_2.x - e_1.x) = 0.$$

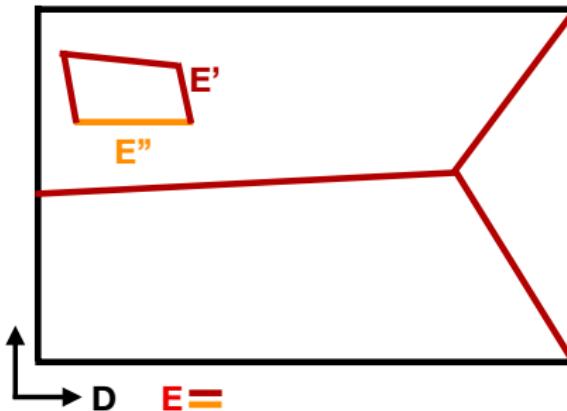


Restrictions (5)

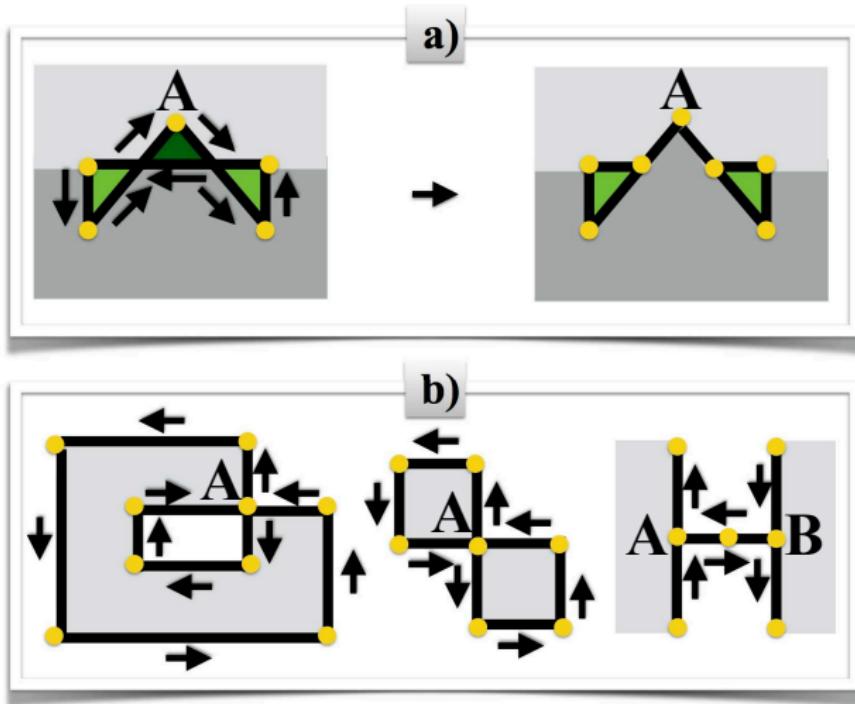
To reduce the degrees of freedom, we only consider edges that are roughly orthogonal to footprint directions. Maximum deviation from $\pm \frac{\pi}{2}$ is determined by the threshold angle α :

$$x_{e,d} = 0 \text{ for all } e \in E', d \in D$$

with
$$\frac{|(e_2.x - e_1.x) \cdot d.x + (e_2.y - e_1.y) \cdot d.y|}{\sqrt{(e_2.x - e_1.x)^2 + (e_2.y - e_1.y)^2}} > \left| \cos \left(\frac{\pi}{2} + \alpha \right) \right|,$$



Elimination of self-intersections¹



¹Quality enhancement techniques for building models derived from sparse point clouds, Proc. GRAPP 2017, pp. 93–104

Results for a city model of a square kilometer

- For $\varepsilon = 1 \text{ m}$, $\alpha = \frac{\pi}{6}$: 1826 non-trivial problem instances for connected components, 61 reach the time limit of two seconds
- For $\varepsilon = 2 \text{ m}$, $\alpha = \frac{\pi}{4}$: 1876 instances, 89 exceed the time limit.

		minimum	maximum	arithmetic mean	median	quartiles
$\varepsilon = 1 \text{ m}, \alpha = \frac{\pi}{6}$	variables	9	2709	66.69	35	20, 45
	binary variables	1	1210	10.34	4	2, 8
	conditions	2	3004	43.61	20	11, 39
	changed vertices	0	233	4.01	3	2, 5
	running time [ms]	0.09	1697.96	17.45	0.46	0.29, 1.28
$\varepsilon = 2 \text{ m}, \alpha = \frac{\pi}{4}$	variables	9	2951	68.05	36	20, 45
	binary variables	1	1500	12.51	4	2, 9
	conditions	2	3679	48.88	22	11, 42
	changed vertices	0	261	4.44	3	2, 5
	running time [ms]	0.12	1765.27	19.08	0.52	0.31, 1.51

Running times are measured on one kernel of a Macbook Pro (2013) with 2.4 GHz i5 processor.

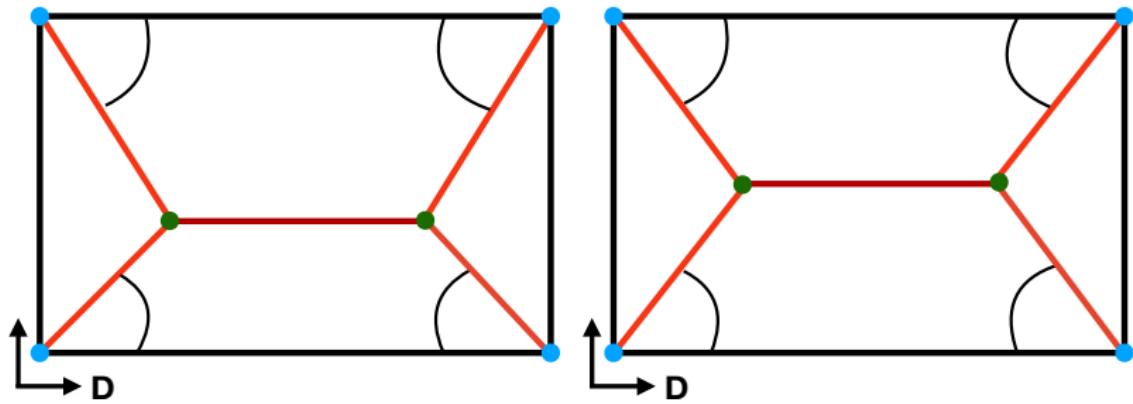
Results ($\varepsilon = 2 \text{ m}$, $\alpha = \frac{\pi}{4}$)



Results ($\varepsilon = 2 \text{ m}$, $\alpha = \frac{\pi}{4}$)



Outlook: Symmetry enhancement



Impressum

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