3D Roof Reconstruction with a Mixed Integer Linear Program

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Outline

- **Model- and data-based generation of 3D city models**
- Estimating planes with RANSAC
- Mixed Integer Linear Program (MILP)
- Results

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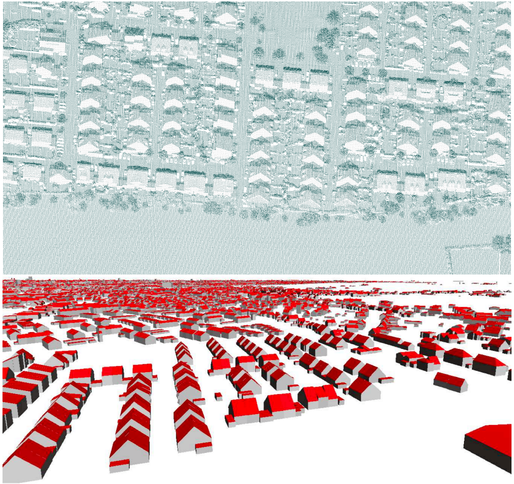
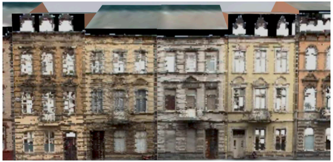
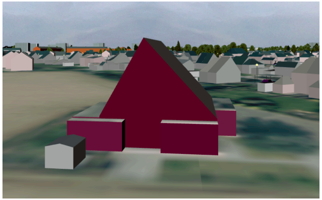
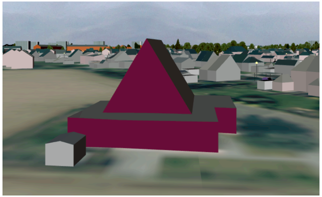
Airborne Laser Scanning point (ALS) cloud and corresponding city model



Deviations of city model roof planes from reality



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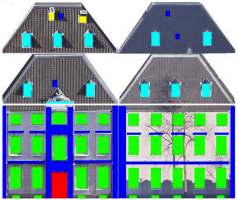
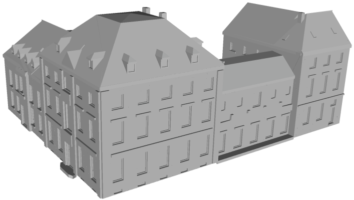
Building shell as a basis for higher levels of detail





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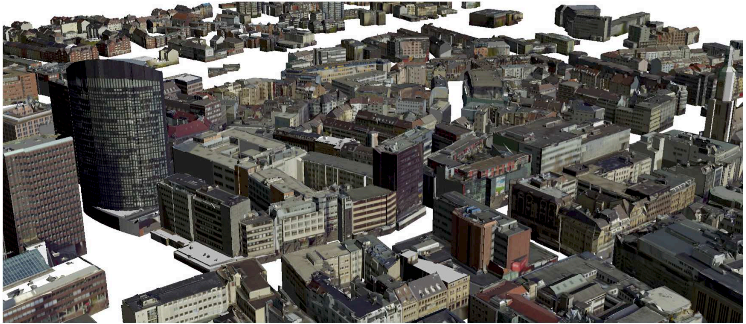

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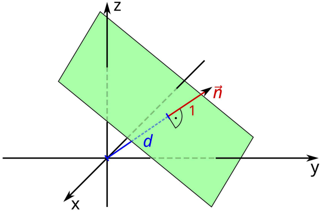
Changing plane equations in an existing city model

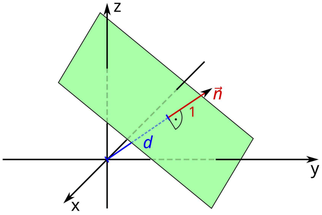


- The points $\vec{p} \in \mathbb{R}^3$ of the plane of roof facet k fulfill the Hessian normal form

$$\vec{p} \cdot \vec{n}_k = d_k$$

where $|d_k|$ is the distance of the plane from the origin $\vec{0}$, and $\vec{n}_k$ is an an (upper) normal with length one.

Changing plane equations in an existing city model



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- We re-estimate the plane with RANSAC to get a new equation

$$\vec{p} \cdot \tilde{\vec{n}}_k = \tilde{\vec{d}}_k$$

  with (upper) normal $\tilde{\vec{n}}_k$, $|\tilde{\vec{n}}_k| = 1$.

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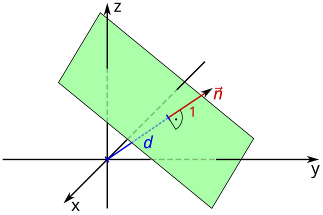

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$$\vec{p} \cdot \tilde{\vec{n}}_k = \tilde{\vec{d}}_k$$

  with (upper) normal $\tilde{\vec{n}}_k$, $|\tilde{\vec{n}}_k| = 1$.
- If the angle between $\vec{n}_k$ and $\tilde{\vec{n}}_k$ is between 2° and 20°, or if the angle is less than 2° but $|d_k - \tilde{d}_k| \geq 10\,\mathrm{cm}$, we use the new plane.

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Random Sample Consensus (RANSAC) to find new equations

procedure RANSAC(P, iteration count i, threshold δ)

$I_{\text{best}} := \emptyset,\ k = 1$

while $(k \leq i) \wedge (|I_{\text{best}}| < |P|)$ **do**

randomly select $\vec{p}_1, \vec{p}_2, \vec{p}_3 \in P$ with $\det[\vec{p}_1, \vec{p}_2, \vec{p}_3] \neq 0$

$(\vec{n}, d) :=$ getPlaneParms($\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$)

I:=getInliers($\vec{n}$, d, P, δ)

if $|I| > |I_{\text{best}}|$ **then**

$I_{\text{best}} := I,\ \vec{n}_{\text{best}} := \vec{n},\ d_{\text{best}} := d$

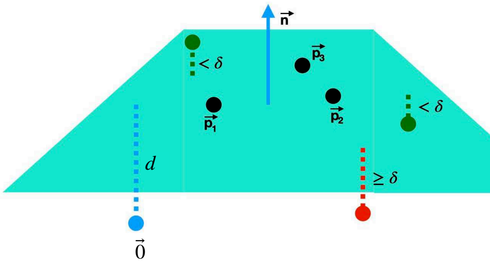
$k := k + 1$

if $|I_{\text{best}}| > 2$ **then**

return $(\vec{n}_{\text{best}}, d_{\text{best}}, I_{\text{best}})$

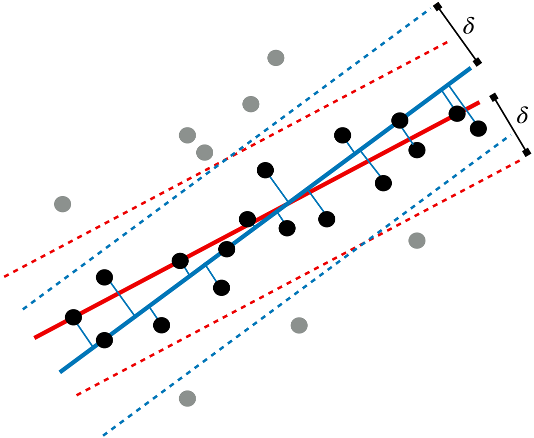
else

return "no plane"



Plane optimization with PCA

We use a Principal Component Analysis to optimally align the RANSAC plane with its inliers.



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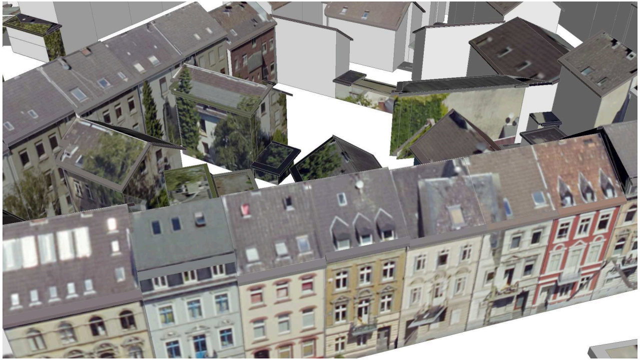

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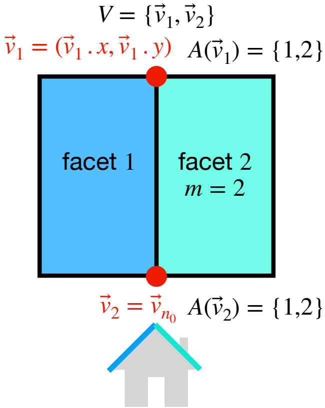
Notations

- Let m be the number of roof polygons of a CityGML building or building part.
- Let $V \subset \mathbb{R}^2$ be the set of all (different) roof polygon vertices with at least two adjacent roof facets projected onto the x-y-plane: $V = \{\vec{v}_1, \ldots, \vec{v}_{n_0}\}$; z-coordinates are handled separately.
- For each vertex $\vec{v}_i = (\vec{v}_i.x, \vec{v}_i.y) \in V$ let

$$A(\vec{v}_i) \subset [m] := \{1, \ldots, m\}$$

be the set of incident roof polygons.
- We map V to $\tilde{V}$, i.e. $\vec{v}_i$ to $\tilde{\vec{v}}_i$ to adjust ridge lines.



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Idea

- Each 2D vertex $\vec{v}_i$, $i \in [n_0]$, must be mapped to $\tilde{\vec{v}}_i$ so that $(\tilde{\vec{v}}_i.x, \tilde{\vec{v}}_i.y, \tilde{z}_i)$ **lies on a maximum number of adjacent planes for a common real z-coordinate $\tilde{z}_i$.**
- Binary variables $b_{k,i}$ indicate whether the 3D vertex $(\tilde{\vec{v}}_i.x, \tilde{\vec{v}}_i.y, \tilde{z}_i)$ lies on the plane with index k, i.e.[1], for all $k \in A(\vec{v}_i)$

$$-M(1 - b_{k,i}) \leq (\tilde{\vec{v}}_i.x, \tilde{\vec{v}}_i.y, \tilde{z}_i) \cdot \tilde{\vec{n}}_k - \tilde{d}_k \leq M(1 - b_{k,i}). \qquad (1)$$

- Thus, a part of the objective function, that has to be maximized, is $\sum_{k \in A(\vec{v}_i)} b_{k,i}$.

[1] For M to be sufficiently large, one has to use a local coordinate system instead of UTM coordinates.

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Keeping changes small

- A mapped 2D vertex must not be too far away from the original vertices. To avoid unnecessary position changes (e.g. on the cadastral footprint) we also minimize such changes as a secondary optimization goal.
- With a threshold value $\delta_0 > 0$ let $0 \leq x_i^+, x_i^-, y_i^+, y_i^- \leq \delta_0$, and

$$x_i^+ - x_i^- = \vec{v}_i.x - \tilde{\vec{v}}_i.x, \;\; y_i^+ - y_i^- = \vec{v}_i.y - \tilde{\vec{v}}_i.y. \tag{2}$$

- Then we extend the objective function to a linear combination:

$$\text{maximize} \left(\sum_{k \in A(\vec{v}_i)} b_{k,i} \right) - \frac{1}{8\delta_0}(x_i^+ + x_i^- + y_i^+ + y_i^-).$$

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Adjusting plane equations to get better results

Variable normals lead to a non-linear problem, but we can vary the distances $\tilde{d}_k$ to $\tilde{d}_k - \varepsilon_k^- + \varepsilon_k^+$ with $\delta_1 > 0$ being a small threshold and $0 \leq \varepsilon_k^-, \varepsilon_k^- < \delta_1$, $k \in [m]$. Then, we optimize globally. Instead of (1), we require that for all $i \in [n_0]$ and $k \in A(\vec{v}_i)$ constraint

$$-M(1 - b_{k,i}) \leq (\tilde{\vec{v}}_i.x, \tilde{\vec{v}}_i.y, \tilde{z}_i) \cdot \tilde{\vec{n}}_k - \tilde{d}_k + \varepsilon_k^- - \varepsilon_k^+ \leq M(1 - b_{k,i}). \tag{3}$$

holds and the global objective is to maximize

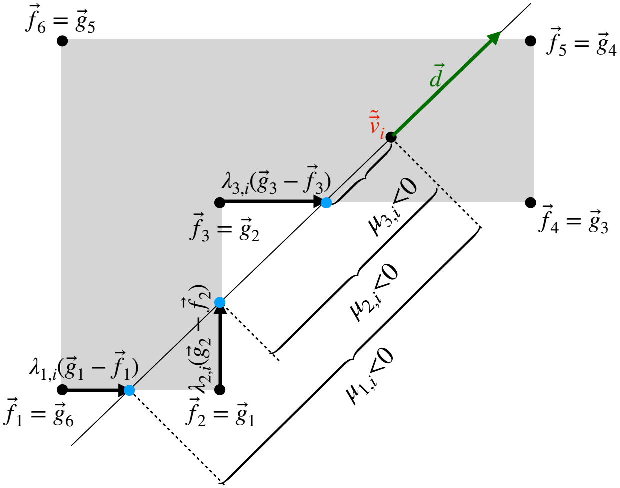
$$\sum_{i=1}^{n_0} \left[\left(\sum_{k \in A(\vec{v}_i)} b_{k,i} \right) - \frac{1}{8 n_0 \delta_0} (x_i^+ + x_i^- + y_i^+ + y_i^-) \right] - \frac{1}{4 m \delta_1} \sum_{k=1}^{m} (\varepsilon_k^- + \varepsilon_k^+)$$

under (2), (3), and following constraints (4)–(12).

Line scan algorithm

- Each vertex $\tilde{\vec{v}}_i$, $i \in [n_0]$ must either be in the interior or on the boundary of the footprint.
- Let $\vec{f}_k$, $\vec{g}_k$ be the endpoints of footprint edges, $k \in [n_1]$.
- A vector $\vec{d} \in \mathbb{R}^2$ with a largest minimum angle with all footprint edges defines the direction of scan lines.
- Intersection of the scan line throught $\tilde{\vec{v}}_i$ with the edge between $\vec{f}_k$ and $\vec{g}_k$:

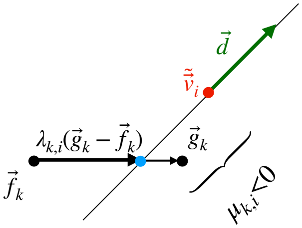
$$\vec{f}_k + \lambda_{k,i} \cdot [\vec{g}_k - \vec{f}_k] = \tilde{\vec{v}}_i + \mu_{k,i}\vec{d}. \quad (4)$$



Checking for intersections with the scan line (1)

$$\vec{f}_k + \lambda_{k,i} \cdot [\vec{g}_k - \vec{f}_k] = \tilde{\vec{v}}_i + \mu_{k,i}\vec{d}.$$

- The intersection is within the edge from $\vec{f}_k$ to $\vec{g}_k$ iff $0 \leq \lambda_{k,i} < 1$.
- We only consider intersections on one side of $\tilde{\vec{v}}_i$ in the sense of $\mu_{k,i} \leq 0$.
- If $\tilde{\vec{v}}_i$ lies on the edge, then also $\mu_{k,i} \geq 0$.



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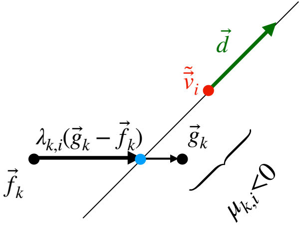

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- We only consider intersections on one side of $\tilde{\vec{v}}_i$ in the sense of $\mu_{k,i} \leq 0$.
- If $\tilde{\vec{v}}_i$ lies on the edge, then also $\mu_{k,i} \geq 0$.

We model these conditions with binary variables $a_{l,k,i}$, $k \in [n_1]$, $l \in [4]$, $M > 0$ large:

$$\lambda_{k,i} < 1 + (1 - a_{1,k,i})M \wedge \lambda_{k,i} \geq 1 - a_{1,k,i}M, \text{ i.e., } \lambda_{k,i} < 1 \iff a_{1,k,i} = 1, \quad (5)$$

$$\lambda_{k,i} \geq -(1 - a_{2,k,i})M \wedge \lambda_{k,i} < a_{2,k,i}M, \text{ i.e., } \lambda_{k,i} \geq 0 \iff a_{2,k,i} = 1, \quad (6)$$

$$\mu_{k,i} \leq (1 - a_{3,k,i})M \wedge \mu_{k,i} > -a_{3,k,i}M, \text{ i.e., } \mu_{k,i} \leq 0 \iff a_{3,k,i} = 1, \quad (7)$$

$$\mu_{k,i} \geq -(1 - a_{4,k,i})M \wedge \mu_{k,i} < a_{4,k,i}M, \text{ i.e., } \mu_{k,i} \geq 0 \iff a_{4,k,i} = 1. \quad (8)$$

Checking for intersections with the scan line (2)

Via linear constraints we set binary variables ($k \in [n_1]$)

$$s_{k,i} := a_{1,k,i} \ \wedge \ a_{2,k,i} \ \wedge \ a_{3,k,i}, \tag{9}$$

$$t_{k,i} := a_{1,k,i} \ \wedge \ a_{2,k,i} \ \wedge \ a_{3,k,i} \ \wedge a_{4,k,i}. \tag{10}$$

- $s_{k,i} = 1 \iff$ the intersection is within the edge before the scan line passes $\tilde{\vec{v}}_i$.
- $t_{k,i} = 1 \iff$ vertex $\tilde{\vec{v}}_i$ lies on the edge.

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Checking for intersections with the scan line (3)

If $\tilde{\vec{v}}_i$ is obtained from a given vertex $\vec{v}_i$ on the footprint, it also has to lie on the cadastral footprint. This leads to the constraint

$$\sum_{k=1}^{n_1} t_{k,i} > 0. \tag{11}$$

On the other hand, if $\vec{v}_i$ is not on the footprint, we have to check with

$$\sum_{k=1}^{n_1} s_{k,i} = 2 \cdot s_i + 1 + t_i, \quad 0 \leq t_i \leq \sum_{k=1}^{n_1} t_{k,i}, \tag{12}$$

$s_i \geq 0$ being an integer, that $\tilde{\vec{v}}_i$ either lies on the footprint (then integer t_i can be chosen to be either 0 or 1) or in its interior (then $\sum_{k=1}^{n_1} s_{k,i}$ has to be odd).

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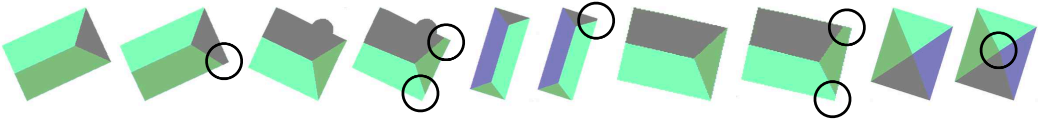
Results

Test with **2.539** buildings or building parts (instances) and corresponding ALS point cloud[2] of square kilometer with southwest UTM coordinates $(330.000, 5.687.000)$:

- **2.144** instances with modified plane equations
- Of these, **1.323** instances with modified flat roofs did not require optimization.
- **817** instances were optimized to optimality with $\delta_0 = \delta_1 = 1\,\mathrm{m}$, $M = 10.000$
- **4** instances had no solution.
- Median running time[3] of MIPs: **0.008 s** ($x_{0.25} = 0.003\,\mathrm{s}$, $x_{0.75} = 0.017\,\mathrm{s}$).
- Median number of vertices: 7, median number of roof facets: 2.



[2] LoD 2 model and point cloud were downloaded from Geobasis NRW on May 24, 2023: `https://www.opengeodata.nrw.de/produkte/geobasis/`

[3] Using the C-API of the IBM CPLEX 22.1.1 optimizer on a laptop with a 2.3 GHz dual-core Intel i5-processor

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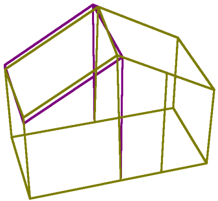
Conclusions

+ A significant number of roof facets in the given city model differ from planes fitted with RANSAC to an ALS point cloud.
+ Only small MIP instances occur, making the optimization approach suitable for application to large urban areas.
– However, the symmetry of the standard roofs is sometimes lost.
– The model-based approach tends to oversimplify. In this case, adjusting the roof gradients is not enough to achieve the correct roof topology.



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