Iterative Closest Point Algorithm for Accurate Registration of Coarsely Registered Point Clouds with CityGML Models

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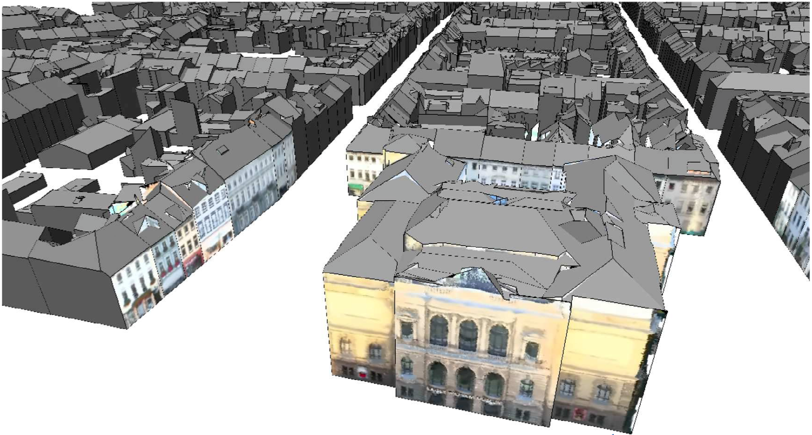
CityGML model, textured with an aligned photogrammetric point cloud



Our aim: Extend LoD2 CityGML models to LoD3.[1]

[1] S. Hensel, S. Goebbels, M. Kada: Facade Reconstruction for Textured LoD2 CityGML Models based on Deep Learning and Mixed Integer Linear Programming, ISPRS GSW 2019

Iterative Closest Point Algorithm (ICP)

Align source point cloud S to target point cloud D.

procedure ICP(point cloud S, point cloud D, convergence criteria)
  while convergence criteria are not fulfilled **do**
    clear correspondences
    for $p \in S$ **do**
      find nearest neighbor $q \in D$ of p
      if neighbor is within threshold distance **then**
        store correspondence (p, q)
    compute affine transform T that maps p to q as close as possible, simultaneously for all pairs (p, q) (least quares minimization)
    apply T to all points in S.

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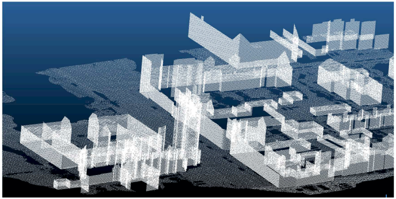
Tests with Point Cloud Library and sampled city models



ICP Singular Value Decomposition	ICP Levenberg-Marquard	ICP Point-to-Plane	NDT	Feature-based alignment

fails | **speed** →

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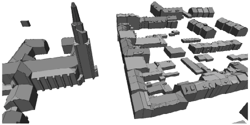
Point-to-Model ICP

Align S:  to D: .

procedure ICP(point cloud S, city model D, convergence criteria)

- **while** convergence criteria are not fulfilled **do**
  - clear correspondences
  - **for** $p \in S$ **do**
    - project p to all wall and roof polygons of D
    - select nearest projection q of p
    - **if** projection is within threshold distance **then**
      - store correspondence (p, q)
  - compute affine transform T that maps p to q as close as possible, simultaneously for all pairs (p, q)
  - apply T to all points in S.

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Residuum of affine transform (homogenous coordinates)

Transformation matrix $T = T_{\vec{a},\sigma}$ depends on a scaling factor σ and parameters $\vec{a} := (\alpha, \beta, \gamma, \Delta x, \Delta y, \Delta z)$.

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For each pair $(\vec{s}, \vec{d})$ we discuss local residuum

$$\vec{r}(\vec{a}, \sigma) = (r_1, r_2, r_3, 0)^\top := T_{\vec{a},\sigma} \cdot \vec{s} - \vec{d} \in \mathbb{R}^4.$$

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We combine all local residua to one vector in $\mathbb{R}^{3n}$:

$$\vec{R}(\vec{a}, \sigma) := (\vec{r}_{1,1}, \vec{r}_{1,2}, \vec{r}_{1,3}, \vec{r}_{2,1}, \ldots, \vec{r}_{n,3})^\top$$

where $\vec{r}_{k,j}$ is the j-th component, $j \in \{1, 2, 3\}$, of the local residuum of pair $(\vec{s}_k, \vec{d}_k)$.

Objective function for computing $\vec{a} = (\alpha, \beta, \gamma, \Delta x, \Delta y, \Delta z)$

Subject to a scaling factor $\sigma = 1$, we have to minimize an objective function (mean of the squared residua)

$$e(\vec{a}, 1) := \left\| \frac{1}{\sqrt{n}} \vec{R}(\vec{a}, 1) \right\|_2^2 = \frac{1}{n} \sum_{k=1}^{3n} (\vec{R}_k(\vec{a}, 1))^2$$

to find optimal rotation and translation parameters.
Scaling parameter σ will be determined in a second step.

Gauss-Newton method

Let $D(\vec{a})$ be the Jacobian of $\frac{1}{\sqrt{n}}\vec{R}(\vec{a},1)$. This matrix of first derivatives can be composed from Jacobians of local residua. Gauss-Newton iteration are defined via $\vec{a}_0 := (0,0,0,0,0,0)$ and

$$\vec{a}_{l+1} := \vec{a}_l - (D(\vec{a}_l)^\top \cdot D(\vec{a}_l))^{-1} \cdot D(\vec{a}_l)^\top \frac{1}{\sqrt{n}}\vec{R}(\vec{a}_l,1).$$

Levenberg-Marquardt method

Let I be the identity matrix and $\lambda_l \geq 0$ parameters controlling the search radius r (step size) for local minima. Initial value: λ_0.

$$\vec{a}_{l+1} := \vec{a}_l - \left(D(\vec{a}_l)^\top \cdot D(\vec{a}_l) + \lambda_l I\right)^{-1} \cdot D(\vec{a}_l)^\top \frac{1}{\sqrt{n}} \vec{R}(\vec{a}_l, 1).$$

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Only downhill steps have to be considered:

- If the objective function does not decrease: parameter λ_l is doubled.
- If the objective function directly decreases: $\lambda_{l+1} := \lambda_l / 2$.

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Remark:

- $\lambda_l = 0$: Gauss-Newton step
- λ_l large: small step of gradient descent.

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Determining a scaling factor σ_0

For previously computed parameters $\vec{a}_l$, minimize

$$e(\sigma) := \sum_{k=1}^{3n} (\vec{R}_k(\vec{a}_l, \sigma))^2.$$

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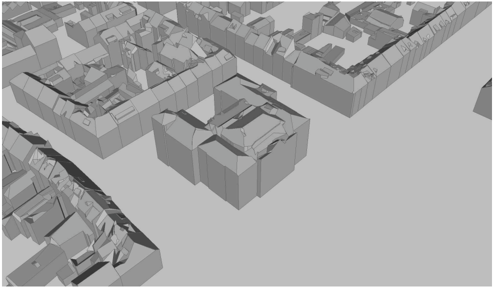
The necessary condition $\frac{d}{d\sigma} e(\sigma) = 0$ results in

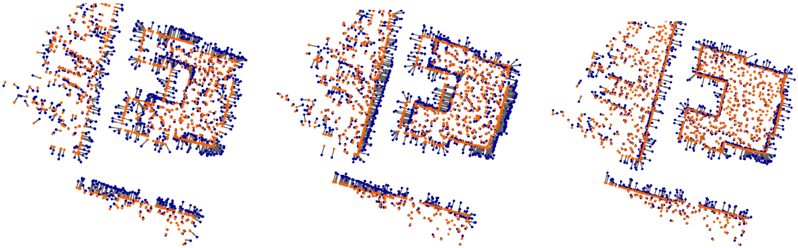
$$\sigma_0 := \frac{\sum_{k=1}^{n} \sum_{j=1}^{3} (T_{\vec{a}_l,1} \vec{s}_k)_j \cdot (\vec{d}_k)_j}{\sum_{k=1}^{n} \sum_{j=1}^{3} (T_{\vec{a}_l,1} \vec{s}_k)_j^2}.$$

Scaling has to be restricted to factors near 1 to avoid contraction of the cloud to one point.

Example: Kaiser Wilhelm museum point cloud



Correspondences at start, after 10, and after 25 iterations



(One Gauss-Newton step, projection to bounding rectangles)

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Levenberg Marquardt optimization (LM) compared with one Gauss-Newton (GN) step

Kaiser Wilhelm museum point cloud: All runs reduce an error of 6.10032 to the same final error 1.976.			
method	number of outer iterations	number of inner iterations	running time
GN	61	limited to 1	326 s
LM $\lambda_0 = 1$	91	limited to 1	488 s
LM $\lambda_0 = 1$	61	average: 4.5	816 s
LM $\lambda_0 = 1/16$	61	average: 2	483 s
LM $\lambda_0 = 1/64$	61	average: 2	484 s

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Projection to roof and wall polygons with gradient descent

- Project point to bounding rectangle.
- If projected point is outside the polygon then move point to polygon's border using gradient descent on a distance transform.

black: section of a polygon

grey value indicates distance

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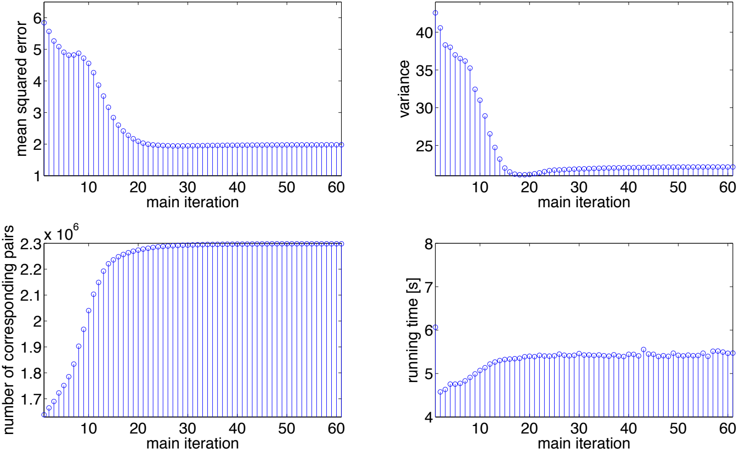
Computational results for projection to rectangles



projection to bounding rectangles (using one Gauss-Newton step, 61 iterations, running time: 326 s)

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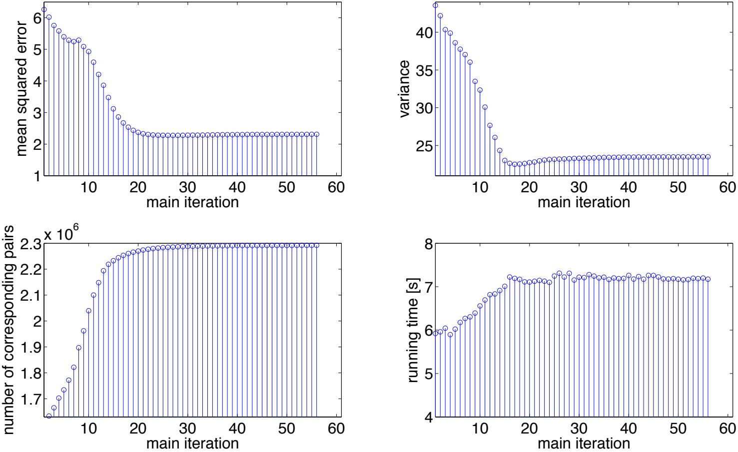
Computational results for projection to polygons



projection to wall and roof polygons (using one Gauss-Newton step, 56 iterations, running time: 392 s)

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