Beautification of City Models based on Mixed Integer Linear Programming

Steffen Goebbels and Regina Pohle-Fröhlich

Niederrhein University of Applied Sciences - Institute for Pattern Recognition, Faculty of Electrical Engineering and Computer Science

OR2018, 12.09.2018
CityGML: Virtual 3D city models
Google Earth/Google Maps 3D
Some applications of city models

- Building Information Modeling (BIM)
- emergency response planning
- solar potential analysis
- urban planning, cadastre
- visualization for navigation and routing
- energy demand estimation
- virtual tours, tourism
- facility management
- flooding simulation
- visibility analysis
- shadow estimation
- noise propagation
- estimation of floor space and population
Laser scanning and photogrammetric point clouds
Previous applications of LP: Planarization of roof facets
Previous applications of MIP: Point cloud registration

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Model vs. data based method

- Model based North Rhine Westphalian city model (©Geobasis NRW 2016)

- Our institute’s data based model
A Problem with data based models: noisy step edges
Solution: Beautification - elementary approach

- Project roof edges to the x-y-plane.
- Compute convex hulls of roof facet’s boundary polygons.
- Compare their area with the area of their smallest enclosing triangles, circles and rectangles.
- If the rectangle area is closest to the convex hull’s area and the difference is below a threshold value, then the contour might be rectangular.
- Replace contour with rectangle if additional heuristic conditions are fulfilled.
Solution: Beautification - optimization approach

- Project roof edges to the $x$-$y$-plane.
- Detect connected components of 2D graph
- Use a MIP to maximize the number of orthogonal edges for each connected component.

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$D$ is a set of normalized 2D direction vectors that are parallel or orthogonal to significant cadastral footprint edges.
The figure shows two connected components that are handled separately.

- For each component, $E$ is the set of all non-trivial 2D edges $e = (e_1, e_2) = ((e_1.x, e_1.y), (e_2.x, e_2.y))$ that do not lie completely on the cadastral footprint.
Notations (3)

- \( E = E' \cup E'' \), edges in \( E' \) are allowed to change their orientation, whereas edges in \( E'' \) have to keep their direction that already is orthogonal to a vector of \( D \).
• $V$ denotes the set of vertices belonging to edges in $E$
• $V' \subset V$ is the set of vertices for which we allow position changes.
• $V'' \subset V'$ is the set of vertices that lie on a footprint edge.
Variables

For $v \in V$, float variables $x_v^+$, $x_v^-$ and $y_v^+$, $y_v^-$ represent non-negative changes. New coordinates are $v.x + x_v^+ - x_v^-$, $v.y + y_v^+ - y_v^-$. For $v \in V \setminus V'$ their values are fixed set to zero.

We introduce binary variables $x_{e,d}$ that indicate, if an edge $e \in E'$ becomes orthogonal to a direction vector $d \in D$:

$$x_{e,d} = \begin{cases} 
1 & : \ e \text{ is transformed to become orthogonal to } d \\
0 & : \ \text{else}
\end{cases}$$
Objective function

- Primary goal: find a maximum number of orthogonal edges
- Secondary goal: minimize coordinate changes.

\[
\text{maximize } \left( \sum_{e \in E'} \sum_{d \in D} x_{e,d} \right) - \frac{1}{8 \cdot |V'| \cdot \varepsilon} \sum_{v \in V'} (x_v^+ + x_v^- + y_v^+ + y_v^-).
\]
Restrictions (1)

We do not change vertices on the cadastral footprint and intersection points of ridge lines:

\[ x_v^+ = x_v^- = y_v^+ = y_v^- = 0 \text{ for all } v \in V \setminus V'. \]

Other vertices can be moved but only within a threshold distance \( \varepsilon > 0 \):

\[ 0 \leq x_v^+, x_v^-, y_v^+, y_v^- < \varepsilon \text{ for all } v \in V'. \]
Restrictions (2)

Modified edges \( e \in E' \) have to be orthogonal to direction \( d \in D \) if \( x_{e,d} = 1 \):

\[
-M \cdot (1 - x_{e,d}) \leq (e_2 \cdot x + x^+_{e_2} - x^-_{e_2} - e_1 \cdot x - x^+_{e_1} + x^-_{e_1}) \cdot d \cdot x \\
+ (e_2 \cdot y + y^+_{e_2} - y^-_{e_2} - e_1 \cdot y - y^+_{e_1} + y^-_{e_1}) \cdot d \cdot y \\
\leq M \cdot (1 - x_{e,d}),
\]

Constant \( M \) is larger than the longest occurring edge.
Restrictions (3)

Vertices in $V''$ are only allowed to change their positions so that they stay on the one footprint edge that they are positioned on: Let $0 \leq r_v \leq 1$ and

$$
\begin{align*}
x_v^+ - x_v^- - (b_v.x - a_v.x) \cdot r_v &= a_v.x - v.x \\
y_v^+ - y_v^- - (b_v.y - a_v.y) \cdot r_v &= a_v.y - v.y,
\end{align*}
$$

where $a_v$ and $b_v$ denote the 2D vertices of the corresponding footprint edge.
Restrictions (4)

We also keep the orientation of edges that are orthogonal to a footprint direction $d \in D$ from the beginning: For all $e \in E''$ let

$\begin{align*}
(x_{e_2}^+ - x_{e_2}^- - x_{e_1}^+ + x_{e_1}^-) \cdot (e_1.y - e_2.y) \\
+ (y_{e_2}^+ - y_{e_2}^- - y_{e_1}^+ + y_{e_1}^-) \cdot (e_2.x - e_1.x) &= 0.
\end{align*}$
Restrictions (5)

To reduce the degrees of freedom, we only consider edges that are roughly orthogonal to footprint directions. Maximum deviation from \( \pm \frac{\pi}{2} \) is determined by the threshold angle \( \alpha \):

\[
x_{e,d} = 0 \text{ for all } e \in E', d \in D
\]

with

\[
\frac{|(e_2.x-e_1.x) \cdot d.x + (e_2.y-e_1.y) \cdot d.y|}{\sqrt{(e_2.x-e_1.x)^2+(e_2.y-e_1.y)^2}} > \left| \cos \left( \frac{\pi}{2} + \alpha \right) \right|
\]
Elimination of self-intersections

1 Quality enhancement techniques for building models derived from sparse point clouds, Proc. GRAPP 2017, pp. 93–104

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Results for a city model of a square kilometer

- For $\varepsilon = 1 \text{ m}$, $\alpha = \frac{\pi}{6}$: 1826 non-trivial problem instances for connected components, 61 reach the time limit of two seconds.
- For $\varepsilon = 2 \text{ m}$, $\alpha = \frac{\pi}{4}$: 1876 instances, 89 exceed the time limit.

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Running times are measured on one kernel of a Macbook Pro (2013) with 2.4 GHz i5 processor.
Results \((\varepsilon = 2 \text{ m}, \ \alpha = \frac{\pi}{4})\)
Results \( (\varepsilon = 2 \text{ m}, \quad \alpha = \frac{\pi}{4}) \)
Outlook: Symmetry enhancement

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Impressum

Prof. Dr. Steffen Goebbels
Niederrhein University of Applied Sciences, iPattern Institute
E-Mail: steffen.goebbels@hsnr.de
Tel.: +49 2151 822 4648

Prof. Dr. Regina Pohle-Fröhlich
Niederrhein University of Applied Sciences, iPattern Institute
E-Mail: regina.pohle@hsnr.de
Tel.: +49 2151 822 4760