# Optimal Student Sectioning at Niederrhein University of Applied Sciences

Steffen Goebbels and Timo Pfeiffer

Niederrhein University of Applied Sciences - Institute for Pattern Recognition,
Faculty of Electrical Engineering and Computer Science

OR 2019

Hochschule Niederrhein
University of Applied Sciences

iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Time table with parallel group sessions

| | 8:00 9:00 | 9:00 10:00 | 10:00 11:00 | 11:00 12:00 | 12:00 13:00 | 13:00 14:00 | 14:00 15:00 | 15:00 16:00 | 16:00 17:00 | 17:00 18:00 | 18:00 19:00 | 19:00 20:00 | 20:00 21:00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Monday** | | | *ENG F* Dj | | *BSY V* Po | | *MAR V* Gp | | *MAR U* Gp | | | | |
| | *OOA P* St | | | | | | | | *OOA T* Shl | | | | |
| | *OOA P* Dv | | | | | | | | | | | | |
| | *BSY P* Ni | | | | | | | | | | | | |
| **Tuesday** | *OOA P* Gb | | | | *ALD V* Ue | | *ALD U* Ue | | *MAR U* Gp | | | | |
| | *BSY P* Ne | | | | | | *TEI2 U* Na | | *BSY U* Po | | | | |
| | *BSY P* Po | | | | | | | | | | | | |
| | *TEI2 P* Na | | | | | | *BSY U* Po | | | *MINT T* ShE | | | |
| | *OOA P* Qi | | | | | | | | | | | | |
| **Wednesday** | *OOA U* St | | *MA2 V* Tp | | *OOA V* St | | *OOA U* St | | *ALD U* Ue | | | | |
| | *MA2 U* Tp | | | | *OOA V* Dv | | *ALD U* Ue | | *OOA U* Dv | | | | |
| | *ENG F* Nn | | | | | | | | | | | | |
| **Thursday** | *TEI2 V* Ha | | *OOA V* St | | *MA2 T* Shl2 | | *ENG F* Ge | | *OOA U* Dv | | *MINT T* Shl | | |
| | | | *OOA V* Dv | | | | *MA2 U* Su | | *MA2 U* Su | | | | |
| | | | | | | | *ALD U* Ue | | | | | | |
| | | | | | | | *ENG F* Db | | | | | | |
| **Friday** | *MA2 V* Tp | | *MA2 U* Tp | | *TEI2 V* Ha | *TEI2 P* Ha | | | | | | | |
| | | | *TEI2 U* Ha | | | | *OOA P* Le | | | | | | |



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Model and Notation

student $s \in [S] := \{1, 2, \ldots, S\}$ is enrolled for a module $k \in [N]$, i.e. $c_{s,k} = 1$

$$b_{k,j,s} = 1$$

| group | $G_{k,1}$ | $G_{k,2}$ | $G_{k,3}$ | $\ldots$ | $G_{k,j}$ | $\ldots$ | $G_{k,n_k}$ |
|---|---|---|---|---|---|---|---|

$$a_{k,j,i,l} = 1$$

| time slot | $T_{k,1}$ | $T_{k,2}$ | $\ldots$ | $T_{k,i}$ | $\ldots$ | $T_{k,m_k}$ |
|---|---|---|---|---|---|---|
| time sub-slot | $T_{k,1,1}$ | $T_{k,2,1}$ | $\ldots$ | $T_{k,i,1}$ | $\ldots$ | $T_{k,m_k,1}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,l}$ | $T_{k,2,l}$ | $\ldots$ | $T_{k,i,l}$ | $\ldots$ | $T_{k,m_k,l}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,p_k}$ | $T_{k,2,p_k}$ | $\ldots$ | $T_{k,i,p_k}$ | $\ldots$ | $T_{k,m_k,p_k}$ |



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Model and Notation

student $s \in [S] := \{1, 2, \dots, S\}$ is enrolled for a module $k \in [N]$,
i.e. $c_{s,k} = 1$

$b_{k,j,s} = 1$

| group | $G_{k,1}$ | $G_{k,2}$ | $G_{k,3}$ | $\dots$ | $G_{k,j}$ | $\dots$ | $G_{k,n_k}$ |
|---|---|---|---|---|---|---|---|

$a_{k,j,i,l} = 1$

| time slot | $T_{k,1}$ | $T_{k,2}$ | $\dots$ | $T_{k,i}$ | $\dots$ | $T_{k,m_k}$ |
|---|---|---|---|---|---|---|
| time sub-slot | $T_{k,1,1}$ | $T_{k,2,1}$ | $\dots$ | $T_{k,i,1}$ | $\dots$ | $T_{k,m_k,1}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,l}$ | $T_{k,2,l}$ | $\dots$ | $T_{k,i,l}$ | $\dots$ | $T_{k,m_k,l}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,p_k}$ | $T_{k,2,p_k}$ | $\dots$ | $T_{k,i,p_k}$ | $\dots$ | $T_{k,m_k,p_k}$ |

# Model and Notation

student $s \in [S] := \{1, 2, \ldots, S\}$ is enrolled for a module $k \in [N]$,
i.e. $c_{s,k} = 1$

$b_{k,j,s} = 1$

| group | $G_{k,1}$ | $G_{k,2}$ | $G_{k,3}$ | $\ldots$ | $G_{k,j}$ | $\ldots$ | $G_{k,n_k}$ |
|---|---|---|---|---|---|---|---|

$a_{k,j,i,l} = 1$

| time slot | $T_{k,1}$ | $T_{k,2}$ | $\ldots$ | $T_{k,i}$ | $\ldots$ | $T_{k,m_k}$ |
|---|---|---|---|---|---|---|
| time sub-slot | $T_{k,1,1}$ | $T_{k,2,1}$ | $\ldots$ | $T_{k,i,1}$ | $\ldots$ | $T_{k,m_k,1}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,l}$ | $T_{k,2,l}$ | $\ldots$ | $T_{k,i,l}$ | $\ldots$ | $T_{k,m_k,l}$ |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $T_{k,1,p_k}$ | $T_{k,2,p_k}$ | $\ldots$ | $T_{k,i,p_k}$ | $\ldots$ | $T_{k,m_k,p_k}$ |

# Restrictions (1)

For each group $G_{k,j}$, binary variables $a_{k,j,i,l}$ are used to assign a time sub-slot:

$$\sum_{i=1}^{m_k} \sum_{l=1}^{p_k} a_{k,j,i,l} = 1. \tag{1}$$

Every sub-slot $T_{k,i,l}$ has to be assigned at most once to a group ($k \in [N]$, $i \in [m_k]$, $l \in [p_k]$):

$$\sum_{j=1}^{n_k} a_{k,j,i,l} \leq 1. \tag{2}$$

Only $q_{k,i}$ groups may be scheduled for a time slot $T_{k,i}$, i.e. ($k \in [N]$, $i \in [m_k]$)

$$\sum_{j=1}^{n_k} \sum_{l=1}^{p_k} a_{k,j,i,l} \leq q_{k,i}. \tag{3}$$

# Restrictions (2)

We use a matrix $C \in \{0,1\}^{S \times N}$ to describe whether a student has enrolled for a module. Thereby, $c_{s,k} = 1$ indicates that student $s$ has chosen module $k$.

Student $s$ is in group $j$ of module $k$ iff $b_{k,j,s} = 1$.

For $k \in [N]$ and $s \in [S]$, we get condition

$$\sum_{j=1}^{n_k} b_{k,j,s} = c_{s,k}. \quad (4)$$

# Time sub-slots

A group of module $k$ assigned to a sub time-slot $T_{k,i,l}$ of time slot $T_{k,i}$ is taught depending on frequency $p_k$ in following weeks:

| frequency $p_k$ | week 1 | week 2 | week 3 | week 4 |
|---|---|---|---|---|
| 1 | $T_{k,i,1}$ | $T_{k,i,1}$ | $T_{k,i,1}$ | $T_{k,i,1}$ |
| 2 | $T_{k,i,1}$ | $T_{k,i,2}$ | $T_{k,i,1}$ | $T_{k,i,2}$ |
| 4 | $T_{k,i,1}$ | $T_{k,i,2}$ | $T_{k,i,3}$ | $T_{k,i,4}$ |

# Time sub-slots

A group of module $k$ assigned to a sub time-slot $T_{k,i,l}$ of time slot $T_{k,i}$ is taught depending on frequency $p_k$ in following weeks:

| frequency $p_k$ | week 1 | week 2 | week 3 | week 4 |
|---|---|---|---|---|
| 1 | $T_{k,i,1}$ | $T_{k,i,1}$ | $T_{k,i,1}$ | $T_{k,i,1}$ |
| 2 | $T_{k,i,1}$ | $T_{k,i,2}$ | $T_{k,i,1}$ | $T_{k,i,2}$ |
| 4 | $T_{k,i,1}$ | $T_{k,i,2}$ | $T_{k,i,3}$ | $T_{k,i,4}$ |

A student can be member of groups of different modules that are taught in overlapping time slots but in different weeks. For example let $p_{k_1} = 2$, $p_{k_2} = 4$, and $p_{k_4} = 4$:

| week 1 | week 2 | week 3 | week 4 |
|---|---|---|---|
| $T_{k_1,i_1,1}$ | | $T_{k_1,i_1,1}$ | |
| | $T_{k_2,i_2,2}$ | | |
| | | | $T_{k_3,i_3,4}$ |

# Time sub-slots with conflicts

Conflicting sub-slots are calculated in advance:

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | X | X | X |

  Let a student be assigned to a group that attends a weekly sub-slot $T_{k_1,i_1,1}$ of module $k_1$.

  $\Longrightarrow$ He cannot attend other time-overlapping sub-slots.



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Time sub-slots with conflicts

Conflicting sub-slots are calculated in advance:

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | X | X | X |

  Let a student be assigned to a group that attends a weekly sub-slot $T_{k_1,i_1,1}$ of module $k_1$.

  $\Longrightarrow$ He cannot attend other time-overlapping sub-slots.

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | | X | |
  | | $T_{k_1,i_1,2}$ | | X |

  A student attends a bi-weekly sub-slot $T_{k_1,i_1,l}$.

  $\Longrightarrow$ He cannot attend an overlapping sub-slot $T_{k_2,i_2,l}$ if it is bi-weekly or four-weekly, and $T_{k_2,i_2,l+2}$ if it is four-weekly.



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Time sub-slots with conflicts

Conflicting sub-slots are calculated in advance:

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | X | X | X |

  Let a student be assigned to a group that attends a weekly sub-slot $T_{k_1,i_1,1}$ of module $k_1$.

  $\Longrightarrow$ He cannot attend other time-overlapping sub-slots.

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | $T_{k_1,i_1,2}$ | X | X |

  A student attends a bi-weekly sub-slot $T_{k_1,i_1,l}$.

  $\Longrightarrow$ He cannot attend an overlapping sub-slot $T_{k_2,i_2,l}$ if it is bi-weekly or four-weekly, and $T_{k_2,i_2,l+2}$ if it is four-weekly.

- | 1 | 2 | 3 | 4 |
  |---|---|---|---|
  | $T_{k_1,i_1,1}$ | $T_{k_1,i_1,2}$ | $T_{k_1,i_1,3}$ | $T_{k_1,i_1,4}$ |

  A student is taught in a four-weekly time sub-slot $T_{k_1,i_1,l}$.

  $\Longrightarrow$ He must not be assigned to overlapping four-weekly sub-slots $T_{k_2,i_2,l}$.

# Restrictions (3)

For all colliding pairs $(T_{k_1,i_1,l_1}, T_{k_2,i_2,l_2})$ of time sub-slots, all groups $j_1 \in [n_{k_1}]$, $j_2 \in [n_{k_2}]$ and all students $s \in [S]$,

$$a_{k_1,j_1,i_1,l_1} + b_{k_1,j_1,s} + a_{k_2,j_2,i_2,l_2} + b_{k_2,j_2,s} \leq 3 \qquad (5)$$

has to be fulfilled.

Further restrictions are used to model rules for part-time and dual education students, car pools, learning groups, etc.



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Objective function: groups of nearly equal size

Difference of sizes of groups $j_1$ and $j_2$ of module $k$ is represented by two non-negative variables $\Delta^+_{k,j_1,j_2}$, $\Delta^-_{k,j_1,j_2} \geq 0$:

$$\Delta^+_{k,j_1,j_2} - \Delta^-_{k,j_1,j_2} = \sum_{s=1}^{S} (b_{k,j_1,s} - b_{k,j_2,s}). \qquad (6)$$

Thus, under above constraints we have to minimize

$$\sum_{k=1}^{N} \sum_{j_1=1}^{n_k-1} \sum_{j_2=j_1+1}^{n_k} (\Delta^+_{k,j_1,j_2} + \Delta^-_{k,j_1,j_2} - \varepsilon_{k,j_1,j_2}). \qquad (7)$$

Slack variables $0 \leq \varepsilon_{k,j_1,j_2} \leq \min\{D, \Delta^+_{k,j_1,j_2} + \Delta^-_{k,j_1,j_2}\}$ allow absolute group size differences to vary between 0 and $D \in \{0, 1, 2, \dots\}$ without penalty.

# Measures to speed-up the program (1)

Certain groups can be assigned to sub-slots in a fixed manner, for example if the number of sub-slots $m_k \cdot p_k$ equals the number of groups $n_k$. For such modules $k$ we can set

$$a_{k,j,i,l} := \begin{cases} 1 & : \quad j = (i-1) \cdot p_k + l \\ 0 & : \quad \text{otherwise.} \end{cases} \tag{8}$$



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Measures to speed-up the program (2)

By assigning groups to sub-slots in a chronologically sorted manner due to their group number, one can avoid many permutations: For all modules $k \in [N]$, all time slots $i_1 \in [m_k]$ and all sub-slots $T_{k,i_1,l_1}$, $l_1 \in [p_k]$, and all groups $j_1 \in [n_k]$ let

$$\begin{aligned} max &\left\{ \sum_{i_2=i_1+1}^{m_k} \sum_{j_2=1}^{j_1-1} \sum_{l_2=1}^{p_k} a_{k,j_2,i_2,l_2}, \sum_{j_2=1}^{j_1-1} \sum_{l_2=l_1+1}^{p_k} a_{k,j_2,i_1,l_2} \right\} \\ &\leq \quad n_k \cdot (1 - a_{k,j_1,i_1,l_1}). \end{aligned} \tag{9}$$

Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Practical application: summer semester 2019

Presented results belong to our bachelor programs in

- computer science (second and fourth semester)
  - 330 students including 59 dual education and part-time students,
  - 30 modules,
  - up to 8 groups per module
- electrical engineering (second, fourth, and sixth semester)
  - 168 students including 40 dual education and part-time students,
  - 27 modules,
  - up to 4 groups per module

# Running times

| slack size $D$ | sorting (9) | init. (8) | computer science | electrical engineering |
|---|---|---|---|---|
| 2 | - | - | 51.59 | 0.13 |
| 2 | - | ✓ | 3.35 | 0.1 |
| 2 | ✓ | - | 2.8 | 0.06 |
| 2 | ✓ | ✓ | 1.57 | 0.05 |
| 1 | - | - | 57.92 | 0.15 |
| 1 | - | ✓ | 6.32 | 0.08 |
| 1 | ✓ | - | 3.49 | 0.09 |
| 1 | ✓ | ✓ | 3.33 | 0.07 |
| 0 | - | - | memory overflow | 2.65 |

IBM ILOG CPLEX processor times (average mean of ten runs) measured in seconds on an Intel Core i5-6500 CPU, 3.20 Ghz x4 with 16 GB RAM.



Hochschule Niederrhein
University of Applied Sciences
iPattern
Institut für Mustererkennung
Institute for Pattern Recognition

# Enhancements

- The program can also be used as an online algorithm for adding late students.
- There might be no feasible solution if students enroll to conflicting modules of different semesters. Such situations can be identified before program execution.
- As a secondary optimization goal, one could maximize the number of students that get commonly assigned to groups along all modules.
- Students who have to repeat modules could be distributed as evenly as possible among groups, since experience has shown that for such students the risk of non-appearance is high.

# Impressum

Prof. Dr. Steffen Goebbels
Niederrhein University of Applied Sciences, iPattern Institute
E-Mail: steffen.goebbels@hsnr.de
Tel.: +49 2151 822 4648



Hochschule Niederrhein
University of Applied Sciences

iPattern
Institut für Mustererkennung
Institute for Pattern Recognition