A 2D Convex Shapes Bin Packing Problem in the Production of Laminated Safety Glass

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Outline

- **The 2D Convex Shapes Bin Packing Problem**
- No-Fit-Polygons
- Mixed Integer Linear Program (MILP)
- Greedy Algorithm
- Results

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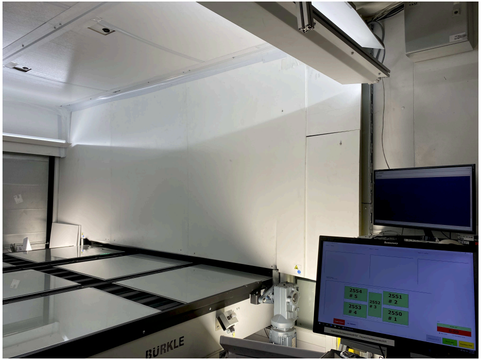
The Bin Packing Problem



- We discuss the nesting of convex polygonal shapes within a minimum number of rectangular spaces (bins) that represent a furnace.
- The problem is motivated by a process of laminating glass tiles supported by the company HEGLA-HANIC GmbH (intelligent glass software).
- The tiles described by the polygons are stacks consisting of glass layers and intermediate foils.

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The Bin Packing Problem (2)

- During the laminating process, a plate is placed from above the tiles with great pressure. The tiles must not be too close to each other, but also not too far apart, so that the pressure does not cause any damage.

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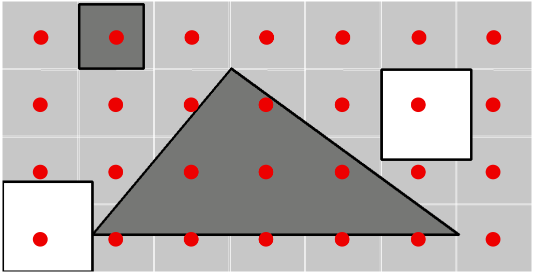
Distance Condition

A feasible one-bin layout for two grey tiles:



- The lower distance bound can be easily achieved by enlarging the polygons through scaling.
- We define a maximum distance bound by requiring that each rectangular region of a grid has to be at least partially covered. Among the solutions that cover a minimum number of bins, a solution with a minimum number of additional support plates (white) is sought.

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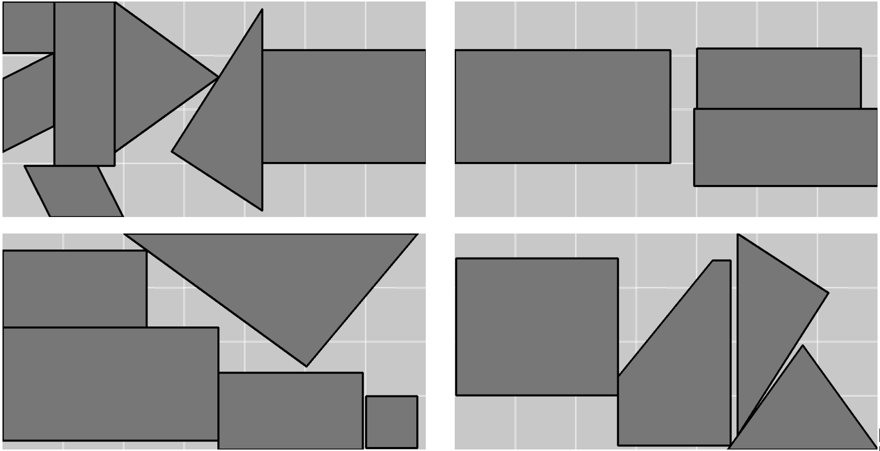
The Bin Packing Problem (3)

- The problem is strongly NP hard: Classical bin packing is reducible to it.
- Example: A problem instance with 19 polygons placed in four bins



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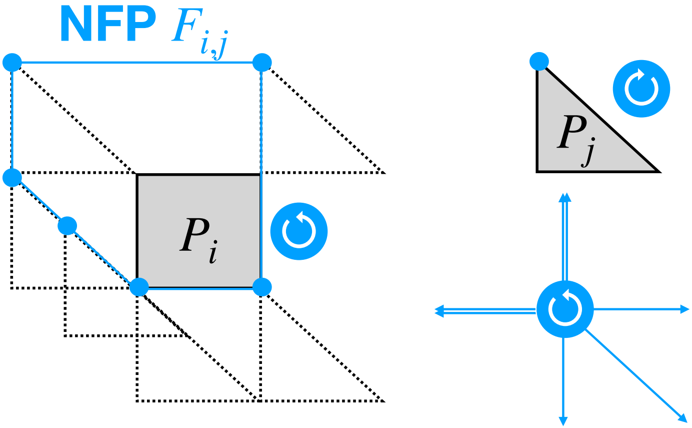
Avoiding Overlaps: No-Fit-Polygons (NFPs)[1]



[1] Cunninghame-Green, R.: Geometry, shoemaking and the milk tray problem. New Sci. 1677, 50–53 (1989)

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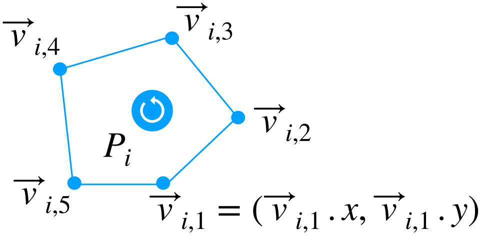
Notations



- Enlarged tiles are represented by convex polygons P_i, $i \in [n] := \{1, \ldots, n\}$. They can be traversed counter-clockwise by following the edges between m_i vertices $\vec{v}_{i,1}, \ldots, \vec{v}_{i,m_i}$ and back to $\vec{v}_{i,m_i+1} := \vec{v}_{i,1}$ where $\vec{v}_{i,k} = (\vec{v}_{i,k}.x, \vec{v}_{i,k}.y) \in \mathbb{R}^2$.
- We also add N support plates P_i, $i \in \{n+1, \ldots, n+N\}$ and a rectangle P_{n+N+1} that defines the distance grid.

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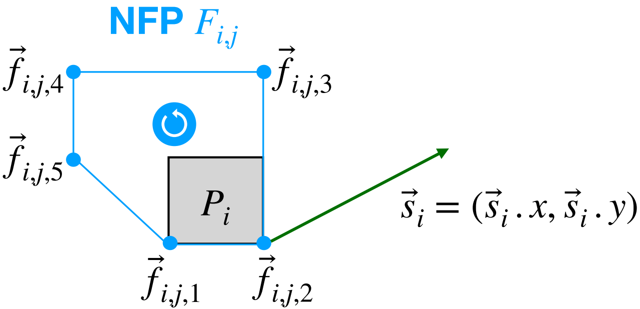
Notations (2)



- For each pair $(i, j) \in [n + N] \times [n + N + 1]$ with $i < j$ we compute an NFP $F_{i,j}$ with vertices $\vec{f}_{i,j,1}, \ldots, \vec{f}_{i,j,m_{i,j}}, \vec{f}_{i,j,m_{i,j}+1} := \vec{f}_{i,j,1}$.
- To shift a polygon to a certain position, we use an offset $\vec{s}_i = (\vec{s}_i.x, \vec{s}_i.y)$. Shifted polygons $\vec{s}_i + P_i$ and $\vec{s}_j + P_j$ do not overlap if and only if $\vec{s}_j + \vec{v}_{j,1}$ lies outside $\vec{s}_i + F_{i,j}$.

Objective Function

Binary variables:

$$x_{i,k} := \begin{cases} 1 & : \text{ polygon } P_i \text{ is placed in bin } k \in [B] \\ 0 & : \text{ otherwise.} \end{cases}$$

$$b_k := \begin{cases} 1 & : \text{ at least one tile is placed in bin } k \in [B] \\ 0 & : \text{ otherwise,} \end{cases}$$

are set via the objective function and constraint

$$\forall_{k \in [B]} \sum_{i \in [n]} x_{i,k} \leq n \cdot b_k.$$

Then the primary goal is to minimize the number of occupied bins, the secondary goal is to use a minimum number of support plates:

$$\text{minimize} \sum_{k \in [B]} b_k + \frac{1}{2N} \sum_{k \in [B]} \sum_{i=n+1}^{n+N} x_{i,k}.$$

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Bin Packing Constraints (1)

- All polygon coordinates have to be within the range of the furnace rectangle.
- Each polygon has to be placed within at most one bin:

$$\forall_{i\in[n+N]} \sum_{k\in[B]} x_{i,k} \leq 1.$$

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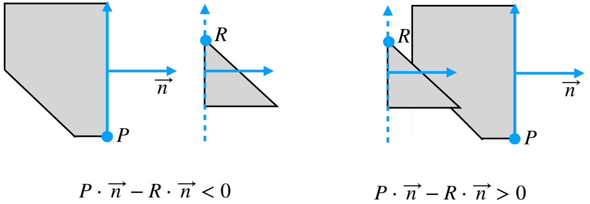
Bin Packing Constraints (2)

There must be no overlaps between polygons $\vec{s}_i + P_i$ and $\vec{s}_j + P_j$ placed within the same bin ($M > 0$ sufficiently large)[2]:

$$\forall_{i,j\in[n+N],i<j}\forall_{k\in[m_{i,j}]}\forall_{l\in[b]} \quad (\vec{f}_{i,j,k+1}.y - \vec{f}_{i,j,k}.y, -\vec{f}_{i,j,k+1}.x + \vec{f}_{i,j,k}.x)\cdot$$
$$\cdot\,[(\vec{s}_i.x + \vec{f}_{i,j,k}.x, \vec{s}_i.y + \vec{f}_{i,j,k}.y) - (\vec{s}_j.x + \vec{v}_{j,1}.x, \vec{s}_j.y + \vec{v}_{j,1}.y)]$$
$$\leq M(2 - x_{i,l} - x_{j,l}) + M(1 - y_{i,j,k}),$$

$$\forall_{i,j\in[n+N],i<j} \sum_{k\in[m_{i,j}]} y_{i,j,k} \geq 1.$$



[2] cf. Leao et al.: Irregular packing problems: A review of mathematical models. Eur. J. Oper. Res. 282(3), 803–822 (2020)

Distance Constraints

Let $\vec{g}_i \in \mathbb{R}^2$, $i \in [g]$, be offset vectors that shift distance rectangle P_{n+N+1} to have a center point at a grid point i.
For each grid point $i \in [g]$ and each bin, at least one intersection between a placed tile or support plate polygon $\vec{s}_j + P_j$ and distance rectangle $\vec{g}_i + P_{n+N+1}$ has to occur. This is indicated by binary variable $z_{i,j,l} = 1$.

$$
\begin{aligned}
&\forall_{i\in[g]}\forall_{j\in[n+N]}\forall_{k\in[m_{j,n+N+1}]}\forall_{l\in[B]}\\
&\qquad(\vec{f}_{j,n+N+1,k+1}.y - \vec{f}_{j,n+N+1,k}.y, -\vec{f}_{j,n+N+1,k+1}.x + \vec{f}_{j,n+N+1,k}.x)\cdot\\
&\qquad\cdot[(\vec{s}_j.x + \vec{f}_{j,n+N+1,k}.x, \vec{s}_j.y + \vec{f}_{j,n+N+1,k}.y)-\\
&\qquad\qquad - (\vec{g}_i.x + \vec{v}_{n+N+1,1}.x, \vec{g}_i.y + \vec{v}_{n+N+1,1}.y)]\\
&\qquad\geq -M(1 - x_{j,l}) - M(1 - z_{i,j,l}),\\
&\forall_{i\in[g]}\forall_{l\in[B]} \sum_{j\in[n+N]} z_{i,j,l} > \sum_{j\in[n+N]} (1 - x_{j,l}).
\end{aligned}
$$

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Rotated Tiles: Orthogonal Rotation

- Only rotations by multiples of 90° (orthogonal rotation) are considered by adding rotated tile polygons (of different shape) to the list of polygons.
- For each index set $I \subset [n]$, representing all rotated instances of a tile, we require

$$\sum_{i \in I} \sum_{k \in [B]} x_{i,k} = 1.$$

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Bottom Left Approach

- We iteratively add tiles to the layout in a given order by using a bottom-left strategy[3].
- Instead of using a local search to find a good order in which to position the tiles, we generate various permutations randomly and select the best outcome. This takes the instability of the problem into account[4].

[3] Junior, B.A., Pinheiro, P.R., Saraiva, R.D.: A hybrid methodology for tackling the irregular strip packing problem. IFAC Proceedings Volumes 46(7), 396–401 (2013).

[4] Lühring, T.: Lösung eines zweidimensionalen Zuschnittproblems für Polygone unter Berücksichtigung zusätzlicher Nebenbedingungen, Master Thesis, Niederrhein University of Applied Sciences, 2022

Bottom Left Approach (2)

- The best outcome is a feasible solution which occupies the smallest number of bins. If there is more than one such solution, a solution is selected that covers the largest number of distance rectangles $\vec{g}_i + P_{n+N+1}$.
- Then, support plates are added.
- For each support plate, we try to shift a tile from the left or from the bottom to fulfill the distance condition without this support plate.

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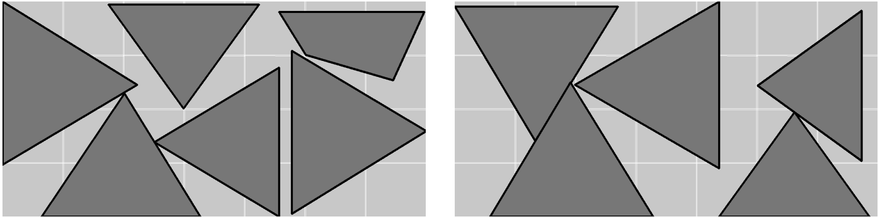
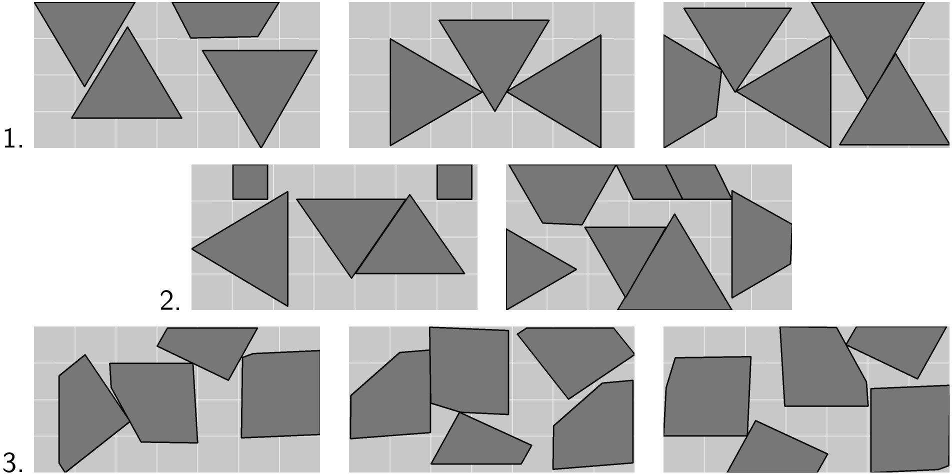
Optimal solution found with CPLEX in 20 s



Feasible solutions found with CPLEX in one hour



Results of the MILP and the greedy strategy

		MILP (CPLEX, 12 threads)			Greedy Strategy (one thread)		
instance	tiles	bins	support	time [s]	bins	support	time [s]
1	19	4	0	exceeded	4	0	13
2	47	–	–	exceeded	9	8 ($\leq$ 5)	86
3	11	2	0	18	2	1 (0)	5
4	12	–	–	exceeded	4	4 ($\leq$ 3)	15
5	12	3	0	exceeded	3	3 (0)	7
6	12	–	–	exceeded	8	7 ($\leq$ 5)	20
7	28	–	–	exceeded	9	3 ($\leq$ 1)	18
8	36	–	–	exceeded	7	5 ($\leq$ 4)	51
9	12	2	0	exceeded	2	0	6
10	14	–	–	exceeded	4	1 (0)	8
11	14	3	0	exceeded	3	1 (0)	12
12	46	–	–	exceeded	16	17 ($\leq$ 9)	73

Feasible solutions found with CPLEX in one hour elapsed time, greedy strategy with 10,000 permutations. Data available at `https://www.hs-niederrhein.de/fileadmin/dateien/FB03/Personen/goebbels/Publikationen/dataset.zip`

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